[Gupta, 6(2): February 2019]
DOI- 10.5281/zenodo.2565939

ISSN 2348 – 8034
Impact Factor- 5.070

GLOBAL JOURNAL OF ENGINEERING SCIENCE AND RESEARCHES

LAPLACE TRANSFORM APPROACH FOR THE HEAT DISSIPATION FROM AN INFINITE FIN SURFACE

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ABSTRACT

In many practical situations, to improve the heat transfer rate extended surfaces, called fins or spines are stuck out from the conducting medium. The amount of heat dissipated from the infinite fin surface into its surroundings can be determined by solving the general differential equation describing one – dimensional heat dissipation from the infinite fin via Laplace transform method. This paper presents a new approach to illustrate the use of Laplace transform in obtaining the rate of heat dissipated into the surroundings from an infinite uniform fin by solving the general form of energy equation describing one–dimensional heat dissipation from the conducting medium. This approach put forward the Laplace transform method as a powerful technique in solving the linear differential equations of second order.

Keywords: *Heat Dissipation, Infinite Fin, Laplace transform, Surroundings.*

I. INTRODUCTION

Fins or spines are the extended surfaces projected from heat conducting surfaces to improve the heat dissipation into the surroundings. Heat transfers by virtue of temperature gradient and the three modes which transfer heat from one part of the medium to another part are conduction, convection and radiation. In conduction mode, heat energy transfers from a part of the medium ata higher temperature to another part of the medium at a lower temperature without any macroscopic motion in the medium. Fourier's law is the basic law of conduction of heat and is expressed as$\mathcal{H} = -\mathcal{K}\mathcal{A}\frac{dt}{dy}$,where $\mathcal{K}$ is the thermal conductivity of the material of the conducting medium, $\mathcal{A}$ is the area of the cross-section of the conducting medium, $\mathcal{H}$is the rate of heat conducted, $\frac{dt}{dy}$is the temperature gradient and the negative sign indicates that the heat is transferring in the direction of decreasing temperature. In convection mode, heat energy transfers from a part of the medium at a higher temperature to another part of the medium at a lower temperature with the macroscopic motion in the medium [1-5].

II. DEFINITION OF LAPLACE TRANSFORMATIONS

Let h(y) is a well-defined function of real numbers y ≥ 0. The Laplace transformation of h(y) is denoted by H (q) or L {h (y)} and is defined as
L {h(y)} $=\int_0^\infty e^{-qy}\, h(y)dy = H(q)$, provided that the integral exists, i.e. convergent. If the integral is convergent for some value of q , then the Laplace transformation of h(y) exists otherwise not. Where q *is* the parameter which may be a real or complex number and L is the Laplace transform operator [6-8].

Laplace transformation of elementary functions:

1. $L\{1\} = \frac{1}{q}$, $q > 0$
2. $L\{y^n\} = \frac{n!}{q^{n+1}}$, $where\ n = 0,1,2,3 \ldots\ldots\ldots$

3. $L\{e^{dy}\} = \dfrac{1}{q-d}$, $q > d$
4. $L\{sindy\} = \dfrac{d}{q^2+d^2}$, $q > 0$
5. $L\{sinhdy\} = \dfrac{d}{q^2-d^2}$, $q > |d|$
6. $L\{cosdy\} = \dfrac{q}{q^2+d^2}$, $q > 0$
7. $L\{coshdy\} = \frac{q}{q^2-d^2}$, $q > |d|$

Laplace Transformation of derivatives:
Let the function h(y) is having an exponential order, that is h(y) is continuous function and is piecewise continuous function on any interval, then the Laplace transform of derivative of h(y) i.e. $L\{h'(y)\}$ is given by

$$L\{h'(y)\} = \int_0^\infty e^{-qy}\, h'(y)dy$$

Integrating by parts and using the condition that, $e^{-qy}\, h(y) = 0\ when\ y = \infty$ we get
$L\{h'(y)\}= [0 - h(0)] - \int_0^\infty -qe^{-qy}\, h(y)dy,$
Or $L\{h'(y)\}= -h(0) + q\int_0^\infty e^{-qy}\, h(y)dy$
Or $L\{h'(y)\}= \quad qL\{h(y)\} - h(0)$
Or $L\{h'(y)\}= \quad qH(q) - h(0)$

Now, since $L\{h'(y)\} = qL\{h(y)\} - h(0),$

Therefore, $L\{h''(y)\} = qL\{h'(y)\} - h'(0)$
Or $L\{h''(y)\} = q\{qL\{h(y)\} - h(0)\} - h'(0)$
Or $L\{h''(y)\} = q^2L\{h(y)\} - qh(0) - h'(0)$
Or $L\{h''(y)\} = q^2H(q) - qh(0) - h'(0)$, and so on.

Inverse Laplace transformation:
The inverse Laplace transform of the function H (q) is L^{-1}[H (q)] or h(y). If we write L [h (y)] = H (q), then L^{-1}[H (q)] = h (y), where L^{-1} is called the inverse Laplace transform operator [6-8].

Inverse Laplace transformations of some functions:

1. $L^{-1}\{\frac{1}{q}\} = 1$
2. $L^{-1}\{\frac{1}{(q-d)}\} = e^{dy}$
3. $L^{-1}\{\frac{1}{q^2+d^2}\} = \frac{1}{d}\, sindy$
4. $L^{-1}\{\frac{q}{q^2+d^2}\} = cosdy$
5. $L^{-1}\{\frac{q}{q^2-d^2}\} = coshdy$
6. $L^{-1}\{\frac{1}{q^2-d^2}\} = \frac{1}{d}\, sinhdy$
7. $L^{-1}\{\frac{1}{q^n}\} = \frac{y^{n-1}}{(n-1)!}$, n > 0.

ISSN 2348 – 8034
Impact Factor- 5.070

III. FORMULATION

Governing differential equation:

The differential equation which describes heat dissipation from an infinite fin can be derived by considering an infinite fin having uniform area of cross-section '$\mathcal{A}$' and perimeter 'Ᵽ'. One end of the infinite fin is connected to a heat source at y = 0 and the other end (i.e. tip of the fin) at y = infinity is free for losing heat into the surroundings. The source of heat is maintained at fixed temperature T. If the temperature of the surroundings of the infinite fin is denoted byt_s and is kept constant, then the convective heat will flow from the infinite fin into the surroundings which lead to a dissipation of heat from its surface into the surroundings [1-5].

Consider an infinitesimal section of thickness Δy of theinfinite fin located at a distance of y from the source R as shown in figure 1.

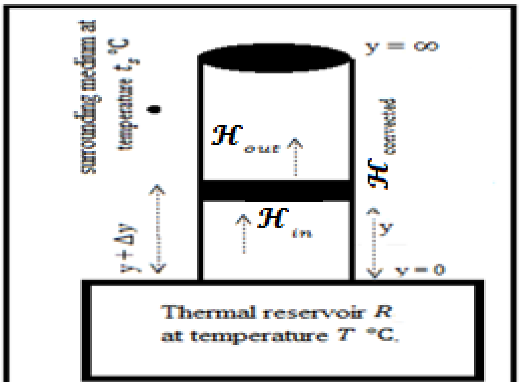

Figure 1: conduction of heat through a infinite fin.

Heat entering into the infinitesimal section is given by

$\mathcal{H}_{in} \;=\; -\mathcal{K}\mathcal{A}\,[D_y t(y)]_y$………. (1) $D_y \equiv \frac{d}{dy}$.

Where t(y) is the temperature of the finite finand is assumed to be constant for the infinitesimal section of the finite fin.

Heat conducted out of the infinitesimal section is given by

$\mathcal{H}_{out} \;=\; -\mathcal{K}\mathcal{A}\,[D_y t(y)]_{y+\Delta y}$

Or

$\mathcal{H}_{out} \;=\; -\mathcal{K}\mathcal{A}\{[D_y t(y) + D_y^2 t(y)]\,\Delta y\}$…. (2)

Heat convected of the infinitesimal section is given by

$\mathcal{H}_{\text{convected}} = \sigma Ᵽ \Delta y[t(y) - t_s]$…. (3)

, where σ is the coefficient of heat transferred by convection mode.

By steady state heat balance, we can write

$\mathcal{H}_{in} \;=\; \mathcal{H}_{out} + \mathcal{H}_{\text{convected}}$…….. (4)

Put equations (1), (2) and (3) in equation (4), we get

$-\mathcal{K}\mathcal{A}[D_y t(y)] = -\mathcal{K}\mathcal{A}\{[D_y t(y) + D_y^2 t(y)]\,\Delta y\} + \sigma Ᵽ \Delta y[t(y) - t_s]$.

Simplifying this equation, we get

$D_y^2 t(y) - \frac{\sigma Ᵽ}{\mathcal{K}\mathcal{A}}[t(y) - t_s] = 0$ ……… (5)

For convenience, let us put

$(\frac{\sigma Ᵽ}{\mathcal{K}\mathcal{A}})^{\frac{1}{2}} = \beta$……. (6)

And define $t(y) - t_s = \tau(y)$ …. (7)

Where $\tau(y)$ is knownexcess temperature at length y of the infinite fin.
Then$D_y[t(y) - t_s] = D_y\tau(y)$.
As t_s is constant, therefore, we can write$D_y^2 t(y)$=$D_y^2 t(y)$,

Therefore, equation (5) can be rewritten as
$D_y^2\tau(y)$ - β^2 $\tau(y)$ = 0 ……….. (8)

Equations (5) and (8) are the general form of energy equations for one-dimensional heat dissipation from the surface of the finite fin.

In equation (6), β isconstant provided thatσ is constant over the entire surface the finite fin and $\mathcal{K}$ is constant within the range of temperature considered.

Solution of the differential equation:
To solve equation (8),the necessary boundary conditions[1-2] are given below:
(i) $t(0)$ = T.In terms of excess temperature, we can write, at y = 0, $t - t_s$ = T- t_s or $\tau(0) = \tau_0$.
(ii) $t(\infty) = t_s$ .In terms of excess temperature, we can write, at y = ∞, $\tau(\infty)$ = 0.

Taking Laplace Transform of equation (8), we get
$L[D_y^2\tau(y)]$ -$\beta^2 L[\tau(y)]$ = 0

This equation gives
$q^2\bar{\tau}$(q) – $q\tau(0)$ -$D_y\tau(0)$- $\beta^2\bar{\tau}$(q) = 0... (9)

Applying boundary condition: $\tau(0) = \tau_0$, equation (9) becomes
$q^2\bar{\tau}$(q) – $q\tau_0$ – $D_y\tau(0)$- $\beta^2\bar{\tau}$(q) = 0
Or
$q^2\bar{\tau}(q) - \beta^2\bar{\tau}(q) = D_y\tau(0) + q\tau_0$….. (10)

In this equation, $D_y\tau(0)$is some constant.
Let us substitute$D_y\tau(0) = \varepsilon$,
Equation (10) becomes
$q^2\bar{\tau}(q) - \beta^2\bar{\tau}(q) = \varepsilon + q\tau_0$
Or $\bar{\tau}(q) = \frac{\varepsilon}{(q^2-\beta^2)} + \frac{q\tau_0}{(q^2-\beta^2)}$ ……. (11)

Taking inverse Laplace transform of above equation, we get
$\tau(y) = \frac{\varepsilon}{\beta} sinhby + \tau_0 \cos h\beta y$
Or
$\tau(y) = \frac{\varepsilon}{2\beta}\left[e^{\beta y} - e^{-\beta y}\right] + \tau_o\left[\frac{e^{\beta y}+e^{-\beta y}}{2}\right]$… (12)

Determination of the constantε:
Applying boundary condition:$\tau(\infty)$ = 0, we can write

$$\frac{\varepsilon}{2\beta}\left[e^{\beta(\infty)} - e^{-\beta(\infty)}\right] + \tau_o\left[\frac{e^{\beta(\infty)} + e^{-\beta(\infty)}}{2}\right] = 0$$

Or $\frac{\varepsilon}{2\beta}\left[e^{\beta(\infty)} - 0\right] + \tau_o\left[\frac{e^{\beta(\infty)}+0}{2}\right] = 0$

Or $\left[\frac{\varepsilon}{2\beta}+\frac{\tau_o}{2}\right]\, e^{\beta(\infty)} = 0$

As $e^{\beta(\infty)} \neq 0$, therefore,

$\left[\frac{\varepsilon}{2\beta}+\frac{\tau_o}{2}\right] = 0$

Or

$\varepsilon = -\beta\tau_o$ …….. (13)

Put the value of ε equation (13) in equation (12), we get

$\tau(y) = \frac{-\beta\tau_o}{2\beta}\left[\, e^{\beta y} - e^{-\beta y}\right] + \tau_o\left[\frac{e^{\beta y}+e^{-\beta y}}{2}\right]$

Or

$\tau(y) = \frac{-\tau_o}{2}\left[\, e^{\beta y} - e^{-\beta y}\right] + \tau_o\left[\frac{e^{\beta y}+e^{-\beta y}}{2}\right]$

Or

$\tau(y) = \frac{\tau_o}{2}\left[\, e^{\beta y} + e^{-\beta y} - e^{\beta y} + e^{-\beta y}\right]$

Or

$\tau(y) = \tau_o e^{-\beta y}$ …….. (14)

Equation (14) provides the expression for the distribution of temperature along the length of the infinite fin and confirms that the temperature of the infinite fin decreases along its length with the increase in distance from the heat source maintained at the temperature$T$.

The amount of heat dissipated from the surface of the infinite fin can be obtained by using the equation

$$\mathcal{H}_f = -\mathcal{KA}\,[D_y t(y)]_{y=0}$$

Or

$\mathcal{H}_f = -\mathcal{KA}\,[D_y \tau(y)]_{y=0}$…… (15)

Now since$D_y\tau(y) = -\beta\tau_o e^{-\beta y}$,

Therefore,

$\left[D_y\tau(y)\right]_{y=0} = -\beta\tau_o$…..(16)

Using equation (16) in equation (15), we get

$\mathcal{H}_f = \mathcal{KA}\beta\tau_o$

Or

$\mathcal{H}_f = \mathcal{KA}\beta(\mathrm{T} - t_s)$….. (17)

Put the value of β from equation (6) in equation (17), we get

$\mathcal{H}_f = \mathcal{KA}\left(\frac{\sigma Ᵽ}{\mathcal{KA}}\right)^{\frac{1}{2}}(\mathrm{T} - t_s)$

Or

$\mathcal{H}_f = (\mathcal{KA}\sigma Ᵽ)^{\frac{1}{2}}(\mathrm{T} - t_s)$ …. (18)

This equation (18) provides expression for the rate of heat transferred from the surface of the infinite fin into its surroundings and confirms that the rate of flow of heat from the infinite fin can be increased by increasing its surface across which the convection of heat occurs.

IV. CONCLUSION

In this paper, an attempt is made to exemplify the Laplace transform approach for determining the temperature distribution along the length of the infinite fin, and the amount of heat dissipated from its surface into the surroundings. This approach brings up the Laplace transform approach as a powerful technique for determining the temperature distribution along the length of the infinite fin, and the amount of heat dissipated from its surface into the surroundings by solving their governing differential equations via the Laplace transform method. We concluded that the temperature of infinite fin decreases with the increase in the length of the infinite fin from the heat source, and the rate of heat transferred from the infinite fin surface into the surroundings can be improved by increasing the surface of the infinite fin across which the convection of heat takes place.

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